

2001

.....3  
 .....4 1  
 .....5 2  
 .....7 2.1  
 .....7 2.2  
 .....12 2.3  
 .....12 2.4  
 .....13 2.5  
 .....13 3  
 .....15 3.1

.....16 3.2  
 .....19 3.3  
 .....19 3.4  
 .....21  
 .....22  
 .....23

X 6 - : Y 1 - ; X 5 - ;  
 ; X 7 - ; X 9 - ;  
 ; X 17 - . 1.  
 . 2.

h ; - : -  
 ; - ,  
 ; - , m (m -  
 A , ; -  
 F , ; -  
 Y F , ; -  
 $\hat{R}$  -  $\hat{R}$  ; -  $\hat{R}$   
 ; - F

: 1)

0,75,

. 2)

X T

X, det(X T X)= 1.425E+6 -

( . 3)

$$M = \frac{\max |\lambda_i|}{\min |\lambda_i|} \lambda_i$$

10).

$$\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \quad M = \frac{2.292}{0.055} = 41.673 \Rightarrow$$

. 4)

$\hat{R}$

$$\hat{R}_1^2 = 0.3849 \quad \hat{R}_2^2 = 0.23232 \quad \hat{R}_3^2 = 0.22113 \quad \hat{R}_4^2 = 0.373154 \quad \hat{R}_5^2 = 0.234$$

$\alpha = 0.05$

$H_0: R_j^2 = 0$ , т.е. коэффициент незначим,

$H_1: R_j^2 \neq 0$

$$F_j = \frac{R_j^2 / (n-1)}{(1-R_j^2) / (N-n-2)} \in \text{Фишера - Снедекера} \left( \frac{n-1}{N-n-2} \right)$$

$F_{\text{max}} > F_{\text{cp}}$ , т.т гипотезу о незначимости отвергаем с вероятностью 0,95  $F_{\text{cp}} = 2.6$

$F_{\text{max}} = 7.51 \quad F_{\text{max}} = 3.63 \quad F_{\text{max}} = 3.41 \quad F_{\text{max}} = 7.14 \quad F_{\text{max}} = 3.67 \quad \dots \quad F_{\text{max}} > F_{\text{cp}}$

$$X_1 / X_2 X_3 X_4 X_5 = \bar{X}_1 + b_2(X_2 - \bar{X}_2) + b_3(X_3 - \bar{X}_3) + b_4(X_4 - \bar{X}_4) + b_5(X_5 - \bar{X}_5)$$

$$X_1 / X_2 X_3 X_4 X_5 = 0.865 - 0.0459X_2 - 0.0109X_3 - 0.000761X_4 - 0.00254X_5 \quad (0,302) \quad (0,524) \quad (0,0003) \quad (0,079)$$

$$X_1 / X_2 X_4 X_5 = 0.865 - 0.0459X_2 - 0.000761X_4 - 0.00254X_5$$

. 2

. 2.1

1, 2, 3, 4, ..., n

Z 1

Z 2

: 1.

, 2.

. K-

Z k (k=1...m)

: 3.

-1

, 4.

$$Z = U'X$$

$$X = UZ, \quad U = (\bar{U}_1 \quad \bar{U}_2 \quad \dots \quad \bar{U}_m)$$

$\bar{Z}_1 \quad \bar{Z}_2 \quad \dots \quad \bar{Z}_m$

$$X = \begin{pmatrix} \frac{X_{11} - \bar{X}_1}{\hat{S}_1} & \dots & \frac{X_{1n} - \bar{X}_1}{\hat{S}_1} \\ \dots & \dots & \dots \\ \frac{X_{m1} - \bar{X}_m}{\hat{S}_m} & \dots & \frac{X_{mn} - \bar{X}_m}{\hat{S}_m} \end{pmatrix}$$

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{N}$$

$$\hat{S}^2 = \frac{1}{N-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

'Stadia'.

$$\hat{R} = \frac{1}{N-1} X \cdot X^T$$

$$\hat{R} = \begin{pmatrix} 1 & -0.069 & 0.415 & 0.167 & -0.940 \\ -0.069 & 1 & -0.219 & -0.034 & 0.146 \\ 0.415 & -0.219 & 1 & 0.028 & -0.389 \\ 0.167 & -0.034 & 0.028 & 1 & -0.172 \\ -0.940 & 0.146 & -0.389 & -0.172 & 1 \end{pmatrix}$$

$$\gamma_n = -\left(N - \frac{1}{6} * (2n+5)\right) \ln |\hat{R}| \chi^2 \frac{n-1}{6}$$

$\gamma_n = 125,7; \chi_{np}^2 (0,05; 3,3) =$

7,8  $\gamma_n \chi_{np}^2$  , 0

$$(x_i x_j) \forall i \neq j \quad 1: \quad (x_i x_j) \neq 0$$

$$\gamma_n = -\left(N - \frac{2n+11}{6}\right) \ln |\hat{R}| \chi^2$$

0

$$\hat{R} \det(R - \lambda E) = 0$$

$$\begin{aligned}\lambda_1 &= 2.292 \\ \lambda_2 &= 1.042 \\ \lambda_3 &= 0.952 \\ \lambda_4 &= 0.659 \\ \lambda_5 &= 0.055\end{aligned}$$

$\lambda_i$

$$\frac{\hat{\lambda}_i}{1+t \sqrt{\frac{2}{N-1}}} \leq \lambda_i \leq \frac{\hat{\lambda}_i}{1-t \sqrt{\frac{2}{N-1}}}$$

$$\begin{aligned}1.656 &\leq \lambda_1 \leq 3.723 & \lambda_1 &= 2.292 \\ 0.753 &\leq \lambda_2 \leq 1.693 & \lambda_2 &= 1.042 \\ 0.688 &\leq \lambda_3 \leq 1.546 & \lambda_3 &= 0.952 \\ 0.476 &\leq \lambda_4 \leq 1.07 & \lambda_4 &= 0.659 \\ 0.04 &\leq \lambda_5 \leq 0.089 & \lambda_5 &= 0.055\end{aligned}$$

$$\chi^2 = (N-1) \sum_{j=1}^2 \ln \hat{\lambda}_j + (N-1) \cdot r \cdot \ln \left( \frac{1}{r} \sum_{j=1}^2 \hat{\lambda}_j \right)$$

$H_0 \chi^2$

$$r = \frac{r(r+1)}{2} - 1$$

$$H_0: \lambda_1 = \lambda_2 \quad \chi^2_{max} = 5.99$$

$$H_1: \lambda_1 \neq \lambda_2 \quad \chi^2_{max} = 93.015 \quad \chi^2_{obs} > \chi^2_{max} H_0$$

$$H_0: \lambda_3 = \lambda_2 \quad \chi^2_{max} = 5.99$$

$$\lambda_1 \quad \lambda_2 \quad H_1: \lambda_3 \neq \lambda_2 \quad \chi^2_{max} = 8.05 \quad \chi^2_{obs} > \chi^2_{max} H_0$$

$$H_0: \lambda_3 = \lambda_4 \quad \chi^2_{max} = 5.99$$

$$\lambda_3 \quad \lambda_2 \quad H_1: \lambda_3 \neq \lambda_4 \quad \chi^2_{max} = 42.815$$

$$\chi^2_{obs} > \chi^2_{max} H_0$$

$\lambda_3 \quad \lambda_4$

0.85.

$$I_k = \frac{\lambda_1 + \lambda_2 + \dots + \lambda_k}{\lambda_1 + \lambda_2 + \dots + \lambda_n} = \frac{\lambda_1 + \dots + \lambda_k}{n} \quad I_1 = \frac{2.292}{5} = 0.458 \quad I_2 = \frac{2.292 + 1.042}{5} \quad I_3 = \frac{2.292 + 1.042 + 0.952}{5} = 0.857$$

$$\lambda \begin{pmatrix} 2.394 & 0 & 0 \\ 0 & 1.45 & 0 \\ 0 & 0 & 0.658 \end{pmatrix}$$

:  $(\hat{R} - \lambda \cdot E) \cdot U^T = 0 \lambda$   
MathCAD,

eigenvec

$$: U = \begin{pmatrix} 0.616 & 0.191 & -0.186 \\ -0.178 & 0.753 & -0.505 \\ 0.418 & -0.376 & -0.09 \\ 0.176 & 0.485 & 0.827 \\ -0.618 & -0.143 & 0.135 \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \quad A = U \cdot \lambda^{y2} = \begin{pmatrix} 0.860 & 0.140 & -0.182 \\ -0.269 & 0.800 & -0.493 \\ 0.633 & -0.384 & -0.088 \\ 0.192 & 0.650 & 0.807 \\ -0.936 & -0.146 & 0.132 \end{pmatrix}$$

$\Rightarrow$

.2.2

$a_{ij}$

i-

j-

(X 5-

) (X 7-

)

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(X 6-

) (X 9-

)

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».2.3

$$\forall j=1, m \quad Z_j$$

$$\frac{Z_j \cdot f_j \cdot Z_j}{\sqrt{\lambda_j}}$$

1,

$$f_j = \lambda_j^{-\frac{1}{2}} \cdot Z_j = \lambda_j^{-\frac{1}{2}} \cdot U_j^T \cdot X$$

$$F = \lambda^{-\frac{1}{2}} \cdot Z = \lambda^{-\frac{1}{2}} \cdot U^T \cdot X \Rightarrow X = U \cdot \lambda^{-\frac{1}{2}} \cdot F$$

$$U \cdot \lambda^{-\frac{1}{2}} = A$$

$$: X_{ij} = a_{i1} \cdot f_{1j} + a_{i2} \cdot f_{2j} + \dots + a_{im} \cdot f_{mj}, \quad X_{ij} I-$$

j-

;  $f_{mj}$ -

j-

;  $a_{im}$ -

I-

$$F = \Lambda^{-1} \cdot A^T \cdot X \quad \Lambda^{-1} \cdot A^T$$

m «

»

.2.4



.1:

.1

.2.5

$$\hat{R}^2_{\min} = \hat{R}^2_{\text{пр}(k)} - 2 \cdot \sqrt{\frac{2k(n-k-1)}{(n-1)(n^2-1)} \cdot (1 - \hat{R}^2(k))}$$

$$\hat{y} = 7.97 - 0.437 f_3$$

“Stadia”,

3.

n

m-

m

$X_i$

$V_i$

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix}$$

$X_i$  $m$ 

$$\left\{ \begin{array}{l} X_1 = a_{11}f_1 + a_{12}f_2 + \dots + a_{1m}f_m + d_1V_1 \\ X_2 = a_{21}f_1 + a_{22}f_2 + \dots + a_{2m}f_m + d_2V_2 \\ \dots \\ X_n = a_{n1}f_1 + a_{n2}f_2 + \dots + a_{nm}f_m + d_nV_n \end{array} \right.$$

 $a_{ij}$  $f_j$  $V_i$  $i$  $d_i$  $i$ 

(1)

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} & \overbrace{0 \ 0 \ \dots \ 0}^n \\ a_{21} & a_{22} & \dots & a_{2m} & 0 \ 0 \ \dots \ 0 \\ \dots & \dots & \dots & \dots & \dots \ \dots \ \dots \ \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} & 0 \ 0 \ \dots \ 0 \end{pmatrix} \quad D = \begin{pmatrix} \overbrace{0 \ 0 \ \dots \ 0}^m & d_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & d_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & d_n \end{pmatrix}$$

$$M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} & d_1 & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & a_{2m} & 0 & d_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} & 0 & 0 & \dots & d_n \end{pmatrix}$$

$$F = \begin{pmatrix} f_{11} & f_{12} & \dots & f_{1N} \\ f_{21} & f_{22} & \dots & f_{2N} \\ \dots & \dots & \dots & \dots \\ f_{m1} & f_{m2} & \dots & f_{mN} \\ v_{11} & v_{12} & \dots & v_{1N} \\ v_{21} & v_{22} & \dots & v_{2N} \\ \dots & \dots & \dots & \dots \\ v_{n1} & v_{n2} & \dots & v_{nN} \end{pmatrix} \quad (1)$$

$$Y = AF \quad (2) \quad 3.1$$

$$\hat{R} = \begin{pmatrix} 1 & -0.069 & 0.415 & 0.167 & -0.940 \\ -0.069 & 1 & -0.219 & -0.034 & 0.146 \\ 0.415 & -0.219 & 1 & 0.028 & -0.389 \\ 0.167 & -0.034 & 0.028 & 1 & -0.172 \\ -0.940 & 0.146 & -0.389 & -0.172 & 1 \end{pmatrix}$$

 $m$  $I$  $a)$ 

$$\hat{R} \quad h_j^2 = \max_{k=1, \dots, n} r_{jk}^2 \quad \frac{2}{h^1} \quad \frac{2}{h^2} \quad \frac{2}{h^3} \quad \frac{2}{h^4} \quad \frac{2}{h^5}$$

$$b) h_j^2 = \frac{1}{n} \sum_{k=1}^n \hat{r}_{jk}^2 \quad (h_1^2, h_2^2, h_3^2, h_4^2, h_5^2)$$

$$h_j^2 = \frac{\hat{r}_{jk} \cdot \hat{r}_{j\bar{k}}}{\hat{r}_{k\bar{k}}} \quad (h_1^2, h_2^2, h_3^2, h_4^2, h_5^2)$$

$$h_j^2 = \frac{\left( \sum_{k=1}^n \hat{r}_{jk} \right)^2}{\sum_{k=1}^n \sum_{\bar{k}} \hat{r}_{k\bar{k}}^2}$$

$R_k$   $h_1^2$   $h_2^2$   $h_3^2$   $h_4^2$   $h_5^2$

$h_1^2$   $h_2^2$   $h_3^2$   $h_4^2$   $h_5^2$

Rh -

$$\hat{R}_k = \begin{pmatrix} 0.398 & -0.069 & 0.415 & 0.167 & -0.940 \\ -0.069 & 0.197 & -0.219 & -0.034 & 0.146 \\ 0.415 & -0.219 & 0.363 & 0.028 & -0.389 \\ 0.167 & -0.034 & 0.028 & 0.152 & -0.172 \\ -0.940 & 0.146 & -0.389 & -0.172 & 0.512 \end{pmatrix}$$

$R_k$

$$\sqrt{\lambda} \bar{a}_1 R^1 = \bar{a}_1 \cdot \bar{a}_1^T R_1 = R_k - R^1 R_1 R_1 \quad \sqrt{\lambda} (R_k)$$

$$: \lambda_1 = 1.658 \quad \lambda_2 = 0.21 \quad \lambda_3 = 0.069 \quad \lambda_4 = -0.105 \quad \lambda_5 = -0.542$$

Rh .

$$\lambda_1 + \lambda_2 + \lambda_3 = 1.897 \quad \text{tr}(R_k) = 1.462$$

$$\lambda_1 = 1.568,$$

$$: \bar{a}_1^* = \begin{pmatrix} 0.631 \\ -0.148 \\ 0.391 \\ 0.148 \\ -0.636 \end{pmatrix} \bar{a}^* |R_k - \lambda E|^* \bar{a}^*$$

$$a_{n1} = \frac{\bar{a}_{n1}^* \sqrt{\lambda_1}}{\sqrt{\sum_{i=1}^n \bar{a}_{ni}^2}} \quad \bar{a}_1 = \begin{pmatrix} 0.812 \\ -0.191 \\ 0.503 \\ 0.191 \\ -0.819 \end{pmatrix}$$

$$\bar{a}_2 =$$



$$\begin{pmatrix} 0.111 \\ 0.324 \\ -0.254 \\ 0.142 \\ -0.088 \end{pmatrix} \lambda_2 = 0,21 \quad \bar{\alpha}_3 = \begin{pmatrix} -0.038 \\ -0.109 \\ -0.034 \\ 0.232 \\ -0.021 \end{pmatrix} \lambda_3 = 0,069$$

$$A = \begin{pmatrix} 0.960 & 0.155 & -0.038 \\ -0.166 & 0.402 & -0.109 \\ 0.826 & -0.409 & -0.034 \\ 0.177 & 0.090 & 0.232 \\ -0.962 & -0.100 & 0.021 \end{pmatrix}$$

(X 5 - )

(X 7 - )

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3.2

$$F = (A^T D^{-1} A)^{-1} \cdot A^T D^{-1} X \quad ; D$$

; X



3.3

$$V_j = \frac{n \sum_{i=1}^n a_{ij}^2 - \left( \sum_{i=1}^n a_{ij} \right)^2}{n^2} \rightarrow \max$$

Vj

Vj : V 1 =0.307, V 2 =0.168



.3:

f1 f2 f1' f2'

= \*

$$\begin{bmatrix} 0.966 & -0.259 \\ 0.259 & 0.966 \end{bmatrix} \begin{bmatrix} 0.969 & -0.099 \\ -0.057 & 0.432 \\ 0.353 & -0.519 \\ 0.195 & 0.041 \\ -0.957 & 0.152 \end{bmatrix} = \begin{bmatrix} \cos 15^\circ & -\sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{bmatrix} \begin{bmatrix} 0.960 & 0.155 \\ -0.166 & 0.402 \\ 0.826 & -0.409 \\ 0.177 & 0.090 \\ -0.962 & -0.100 \end{bmatrix} *$$

1 = 0,240, Vj = 0,156. Vj : V

Vj

«Stadia».

i-

$$\hat{y} = 7.97 + 0.309 f1 + 0.0722 f2 + 0.186 f3$$

1  
1998.- 352 . 2  
1999.-598 .

1

Y1	X5	X6	X7	X9	X17
9,26	0,780,4	1,370,23	17,72		
9,38	0,750,26	1,490,39	18,39		
12,11	0,680,4	1,440,43	26,46		
10,81	0,7	1,420,18	22,37		
9,35	0,620,4	1,350,15	28,13		
9,87	0,760,19	1,390,34	17,55		
9,17	0,730,25	1,160,38	21,92		
9,12	0,710,44	1,270,09	19,52		
5,88	0,690,17	1,160,14	23,99		
6,3	0,730,39	1,250,21	21,76		
6,22	0,680,33	1,130,42	25,68		
5,49	0,740,25	1,1	0,05	18,13	
6,5	0,660,32	1,150,29	25,74		
6,61	0,720,02	1,230,48	21,21		
4,32	0,680,06	1,390,41	22,97		
7,37	0,770,15	1,380,62	16,38		
7,02	0,780,08	1,350,56	13,21		
8,25	0,780,2	1,42	1,76	14,48	
8,15	0,810,2	1,37	1,31	13,38	
8,72	0,790,3	1,410,45	13,69		
6,64	0,770,24	1,350,5	16,66		
8,1	0,780,1	1,480,77	15,06		
5,52	0,720,11	1,24	1,2	20,09	
9,37	0,790,47	1,4	0,21	15,98	
13,17	0,770,53	1,450,25	18,27		
6,67	0,8	0,34	1,4	0,15	14,42
5,68	0,710,2	1,280,66	22,76		
5,22	0,790,24	1,330,74	15,41		
10,02	0,760,54	1,220,32	19,35		
8,16	0,780,4	1,280,89	16,83		
3,78	0,620,2	1,470,23	30,53		
6,48	0,750,64	1,270,32	17,98		
10,44	0,710,42	1,510,54	22,09		
7,65	0,740,27	1,460,75	18,29		
8,77	0,650,37	1,270,16	26,05		
7	0,660,38	1,430,24	26,2		
11,06	0,840,35	1,5	0,59	17,26	
9,02	0,740,42	1,350,56	18,83		
13,28	0,750,32	1,410,63	19,7		

9,27 0,750,331,471,1 16,87  
6,7 0,790,291,350,3914,63  
6,69 0,720,3 1,4 0,7322,17  
9,42 0,7 0,561,2 0,2822,62  
7,24 0,660,421,150,1 26,44  
5,39 0,690,261,090,6822,26  
5,61 0,710,161,260,8719,13  
5,59 0,730,451,360,4918,28  
6,57 0,650,311,150,1628,23  
6,54 0,820,081,870,8512,39  
4,23 0,8 0,681,170,1311,64  
5,22 0,830,031,610,498,62  
18 0,7 0,021,340,0920,1  
11,030,740,221,220,7919,41

	f1	f2	f3
1	0.465	0.513	-0.722
2	0.521	-0.576	-0.18
3	-0.918	-0.263	-0.119
4	-0.53	0.434	-0.672
5	-1.703	-0.315	0.16
6	0.527	-0.593	0.05
7	-0.574	0.059	0.243
8	-0.455	0.651	-0.508
9	-1.005	-0.546	0.676
10	-0.495	0.48	-0.315
11	-1.401	0.233	0.292
12	-0.293	0.333	0.082
13	-1.516	0.049	0.366
14	-0.277	-1.222	0.996
15	-0.456	-1.647	0.942
16	0.722	-0.662	0.164
17	1.067	-0.793	0.279
18	1.029	-0.334	0.062
19	1.246	-0.106	-0.118
20	1.05	0.109	-0.534
21	0.569	-0.175	-0.127
22	1.149	-1.072	0.215
23	-0.212	-0.722	0.771
24	0.698	0.853	-1.066
25	0.399	0.874	-1.153
26	1.007	0.311	-0.723
27	-0.523	-0.562	0.473
28	0.797	6.03E-3	-0.184
29	-0.225	1.458	-0.957
30	0.382	0.833	-0.584
31	-1.525	-1.642	0.833
32	-0.161	1.809	-1.328
33	-0.185	-0.104	-0.45
34	0.395	-0.45	-0.103
35	-1.426	-0.081	0.145

36-1.057-0.412 -0.012  
 37 1.263 0.194 -0.811  
 38 0.016 0.516 -0.546  
 39 0.211 -0.1 -0.251  
 40 0.576 -0.082 -0.332  
 41 1.703 3.644 5.731  
 42 -0.235-0.339 0.019  
 43 -1.023 1.293 -0.705  
 44 -1.656 0.487 0.022  
 45 -1.047 0.164 0.457  
 46 -0.211-0.573 0.546  
 47 -0.017 0.608 -0.645  
 48 -1.804-0.119 0.487  
 49 2.464 -1.953 -0.182  
 50 0.543 2.607 -1.793  
 51 2.391 -1.4 -0.05  
 52 -0.127-1.581 0.901  
 53 -0.131-0.094 0.26

2 3  
 . : . .std =2 2  
 . f1 3,77E-5 1 f2 5,66E-7 1 f3 3,77E-5 1 Y 7,97 2,61  
 =0,57 f1 f2 f3 Y f2 0 f3 -0,001 0 Y 0,044 0,009 -0,167 e  
 =0 (0%) \*\*\* No.1,  
 :f3 . a0 a1 7,97 -0,437 . . 0,357 0,36 . 0 0,229